U.G. 4th Semester Examination - 2022

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-4

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

Answer all the questions.

1. Answer any **ten** questions: 2×10=20

a) In a Group (G, o) Prove that $(a^{-1})^{-1} = a$ for all $a \in G$.

b) If a Group (G, ;) be abelian, show that $(ab)^2 = a^2 b^2$ for all $a, b \in G$.

c) Show that cube roots of unity is a cyclic group.

d) Prove that a Group (G, o) contains only one identity element.

e) Give example of a finite non abelian group.

f) Define order of a group. Give an example of a non cyclic group.

g) Show that a cyclic group is necessarily abelian.

h) Define a normal subgroup. Give an example, in non commutative case.

i) Define Quotient group. Give an example.

j) In a ring $(R,+,\times)$ with 0 as additive identity, prove that $a\times(-b) = (-a)\times b = -(a\times b)$.

k) Define Characteristic of a Ring. Find the Characteristic of the Ring $(Z,+,\times)$.

l) Prove that intersection of two subrings is a subring.

m) Prove that in a ring (R, +, .) if a is an identity element then 1–a is also an identity element.

n) If a,b be two elements of a group (G, o) and (H, o) be its subgroup. Then prove that $b \in aH$ implies $a^{-1} b \in H$.

o) Prove that the order of a cyclic group is equal to the order of its generators.

2. Answer any **four** of the following questions : 5×4=20

a) Prove that Z_n,the classes of residues of integers modulo n forms an abelian group.

b) Prove that union of two subgroups of a group (G, o) is a subgroup iff any one of those is contained in the other.

c) Prove that every group of prime order is cyclic.

d) Show that the set , $S=\{a+b\sqrt{2}; a,b \in Z\}$ forms a ring with respect to addition and multiplication.

e) Prove that in a ring $a^2-b^2=(a+b)(a-b)$ is not in general true.

f) Prove that the group $\{1,-1, i,-i\}$ is cyclic and finds its generators.

g) Prove that every field is an integral domain.

3. Answer any **two** of the followings : $10\times2=20$

i) a) Prove that a subgroup H of a group G is normal iff $h\in H$, $x \in G$ implies $xhx^{-1}\in H$.

b) If G be a group and for all $a,b\in G$, $a^4=e$ and $a^2 b = b a$, then show that $a = e$.

c) Prove that a finite integral domain is a field. 4+3+3

ii) a) If H be a subgroup of a commutative group G then prove that quotient group G/H is commutative.

b) Prove that every non zero element in a finite ring having no devisor of zero is a unit.

c) Prove that every proper subgroup of a group of order 6 is cyclic. 3+3+4

iii) a) Prove that every subgroup of a cyclic group is cyclic.

b) Prove that if $a^2=e$ for all a in a group G, then G is abelian.

c) Let $(R,+,.)$ be a ring and S be a non-empty subset of R , then S is a subring of R iff $a,b \in S$ implies $a-b \in S$ and $a.b \in S$.

3+3+4

iv) a) For all a,b in a group (G, .), if $(a.b)^3=a^3.b^3$ and $(a.b)^5=a^5.b^5$, then show that the group is abelian.

b) Show that the set of all positive rational numbers with respect to binary composition $\circ$ defined by $a \circ b = ab/2$ forms a group.

c) If a be unit in a ring R prove that its multiplicative inverse is unique.

4+3+3